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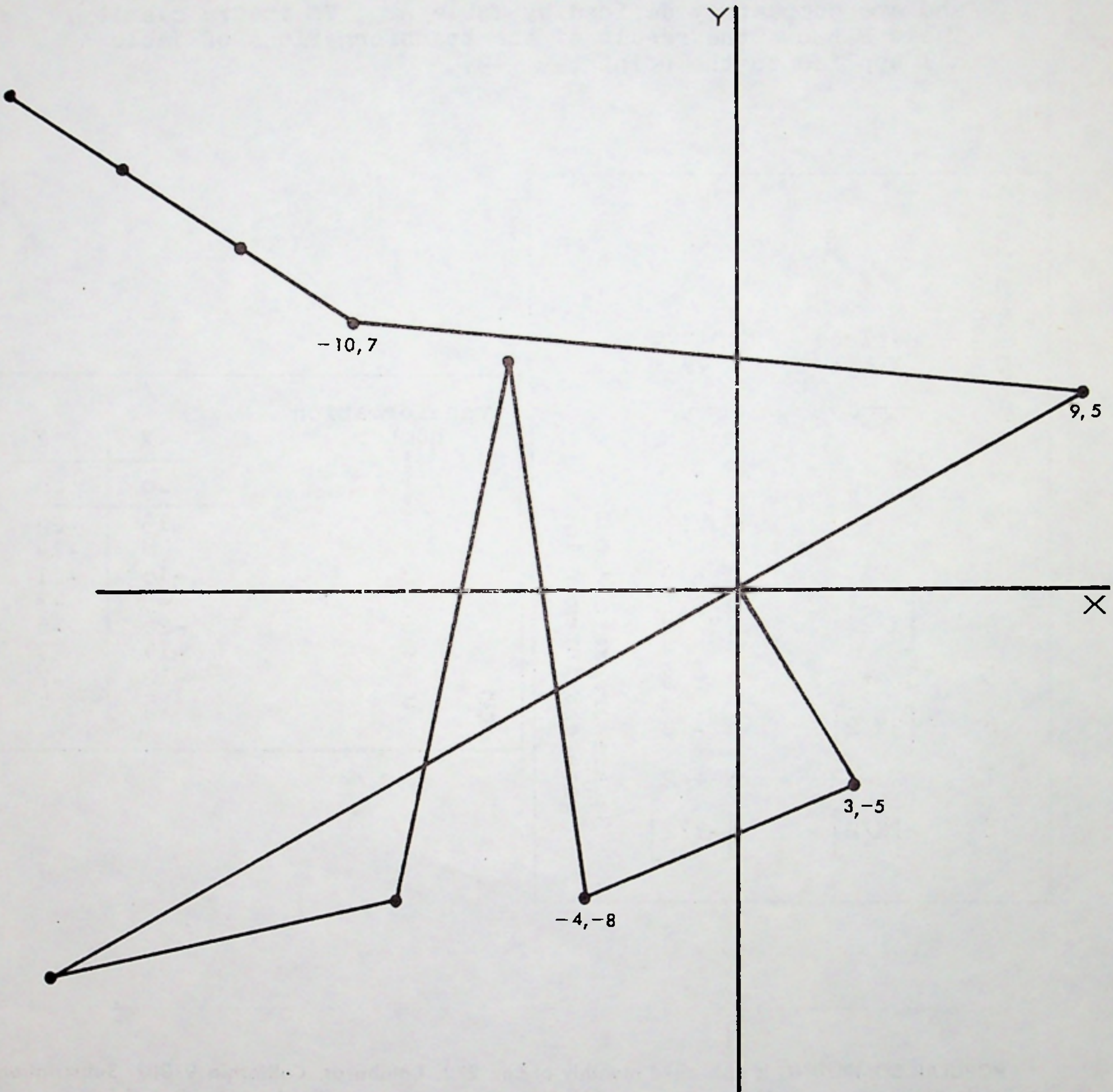

Change of Base

PROBLEM 46

Change of Base

In previous "trip" Problems (The Pi Dragon, The Road to e, The Web of Fibonacci), the computing part of the problem consisted of the moves to be made on the coordinate grid.

In this problem, the moves themselves are simple, and are completely defined by Table A. To insure clarity, Table B shows the result of the transformations of Table A all applied to the point (-9, -9).

A

Transformation number	Replace X by:	Replace Y by:
1	2X + 3	2Y - 5
2	-X - 1	[3Y/2]
3	2X	Y - 2
4	[X/2]	[Y/3]
5	3X + 1	Y + 3
6	[3X/2]	-Y - 2
7	X - 3	Y + 2
8	-[X/2]	[-Y/2]

Square brackets denote "greatest integer in."

B

Transformation number	X	Y
	-9	-9
1	-15	-23
2	8	-14
3	-18	-11
4	-5	-3
5	-26	-6
6	-14	7
7	-12	-7
8	5	4

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Table C (page 4) is a table of factorials, in base 9 notation. The circled numbers are the low order, non-zero digits. The sequence of those digits dictates the moves for this trip. That is, we are to apply transformations 1, 2, 6, 6, 3, and so on, to X and Y coordinates, starting at the origin. Thus we have:

Transformation number	X	Y	
	0	0	START
1	3	-5	
2	-4	-8	
6	-6	6	
6	-9	-8	
3	-18	-10	
8	9	5	
2	-10	7	
7	-13	9	
7	-16	11	

The first ten moves of the trip are shown graphically on the cover. The Problem is this: Where is the 200th point?

The real problem, then, is the calculation of the low order non-zero digits of the factorials in base 9. Table C shows the first 20 of these. Table D shows the digits from 101 through 200. The digits from 21 through 100 are to be calculated, and all 200 digits used to dictate the transformations for the trip.

D Low order non-zero digits of the factorials, base 9; digits for positions for 101! through 200!

43367 45663 47833 66637 15336 33685 76632 24336 74588 73361
72775 66325 48873 36172 22433 67451 12663 82733 68576 63336

1	1
2	2
3	6
4	2 6
5	1 4 3
6	8 8 0
7	6 8 2 0
8	6 1 2 7 0
10	6 1 2 7 0 0
11	6 7 4 0 7 0 0
12	8 3 0 8 8 5 0 0
13	1 2 1 1 2 8 3 6 0 0
14	1 7 0 5 8 2 1 2 6 0 0
15	2 7 0 0 1 7 3 0 1 3 0 0
16	4 5 6 0 3 0 2 0 2 2 0 0 0
17	8 2 0 6 5 3 3 5 3 8 5 0 0 0
18	1 6 4 8 3 4 1 3 7 2 4 1 4 0 0 0
20	3 4 0 7 6 8 2 7 5 4 8 2 8 0 0 0 0

fig. C

A Merging Problem

PROBLEM 47

Given four blocks of storage as follows:

Ten words addressed at A through A+9.
Ten words addressed at B through B+9.
Ten words addressed at C through C+9.
Thirty words addressed at D through D+29.

Blocks A, B, and C contain numbers which are in ascending order within each block; there are no duplicates among these 30 numbers. We want to merge the 30 numbers into block D. (It would be feasible to simply move all 30 numbers into block D and then sort block D, but this would be inefficient.)

This is to be a subroutine. The main routine has already verified that blocks A, B, and C are as stated, so the subroutine need not edit the data.

A) Draw a flowchart of the logic involved.

B) Outline a procedure to test a debugged program that follows the logic of that flowchart.

There is shown below a set of sample data, to insure that the situation is clear, but of course the logic must apply to any data that fits the given conditions.

13	14	15	28	35	57	128	350	600	1000
A	A+1	A+2	A+3	A+4	A+5	A+6	A+7	A+8	A+9

1	16	50	51	52	300	400	500	991	999
B	B+1	B+2	B+3	B+4	B+5	B+6	B+7	B+8	B+9

10	12	20	40	60	80	81	82	83	1001
C	C+1	C+2	C+3	C+4	C+5	C+6	C+7	C+8	C+9

Roots to Order

In the study of numerical methods, it is expedient to have equations at hand for which various algorithms for finding roots may be applied. What is wanted are polynomials of not too high degree, with integral coefficients that are fairly small, and having irrational roots. For pedagogical reasons, the roots should be easy to predict by the instructor.

Consider the possibilities:

1. Use quadratics; the roots can be checked by formula. The roots can also be found by formula, and a student may properly wonder what we're doing. The degree is too small.

2. Fabricate an equation by building up linear factors, such as:

$$(x - 3)(x + 7/2)(x - 5) = 0.$$

Such an equation will either have rational roots, or, if it has irrational roots, then its coefficients will also be irrational.

3. Combine the first two methods, as in:

$$(x^2 + 4x - 7)(x - 3) = 0$$

where the left factor has zeros at $(-2 \pm \sqrt{11})$. But the third root, again, is rational, which spoils the problem.

4. Use stock equations for which the roots have been calculated, such as Wallis' equation:

$$x^3 - 2x - 5 = 0$$

for which the real root is known to some 2000 digits.

5. Make up an equation with known rational roots and then translate it vertically so that the roots become irrational. For example, the equation:

$$x^3 - 5x^2 - 29x + 105 = 0$$

has roots at 3, -5, and 7. Thus, the equation

$$x^3 - 5x^2 - 29x + 104 = 0$$

has roots that are near 3, -5, and 7 and are irrational. (The smallest root is 2.9689.)

6. Work the problem backwards. Cardan's formulas solve the cubic analytically, so we can work from the inside out. The critical part of the formulas calls for the value of:

$$R = \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}$$

so we can pick values of p and q to make that term rational. For example, if q is 6 and p is 9, the radical has the value 6. Then, for

$$A = (-q/2) + R$$

$$B = (-q/2) - R$$

a root of the cubic $x^3 + px + q = 0$ is:

$$\sqrt[3]{A} + \sqrt[3]{B}$$

For the example given, we have:

$$\sqrt[3]{3} + \sqrt[3]{-9}$$

which can be readily calculated from a table of cube roots:

$$-.6378342527444957322$$

for the equation $x^3 + 9x + 6 = 0$.

The equation can be translated by replacing x by x + k. For example, if x-3 replaces x, we have:

$$x^3 - 9x^2 + 36x - 48 = 0$$

for which the roots are 3 greater than the original, or

$$x = 2.362165747255504.$$

Gauss's Lattice Problem

PROBLEM 48

This problem is expounded in a booklet, "Lattice Points in a Circle; Experiments and Conjecture," by M. E. Rose, Computing and Mathematics Curriculum Project, University of Denver, Department of Mathematics, Denver, Colorado 80210.

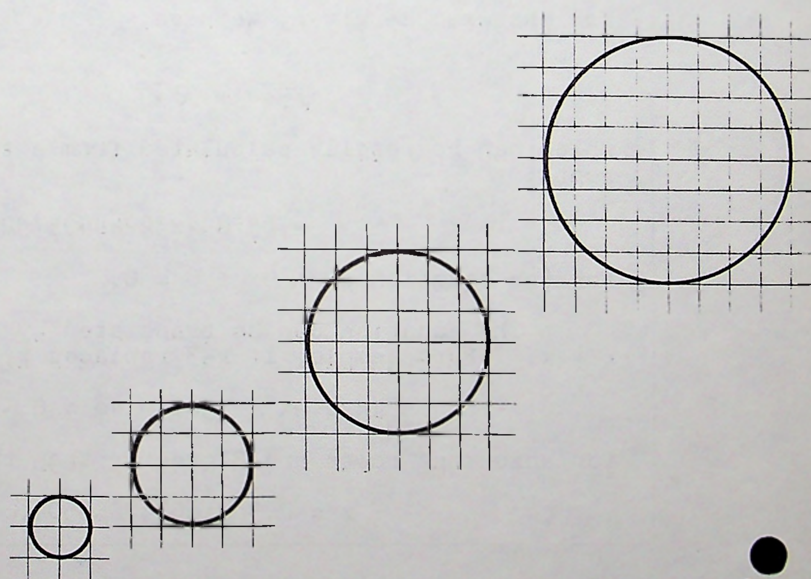
The problem is this: how many points of a lattice are in or on a circle of radius R centered at the origin? The Figures show the cases for R = 1, 2, 3, and 4, for which the count of points in or on each circle is 5, 13, 29, and 49 respectively. It was a conjecture of Gauss' that it is not possible to write a formula for the number of points, Q, as a function of R. For a given R, the number Q can be counted by finding all values of X and Y that satisfy

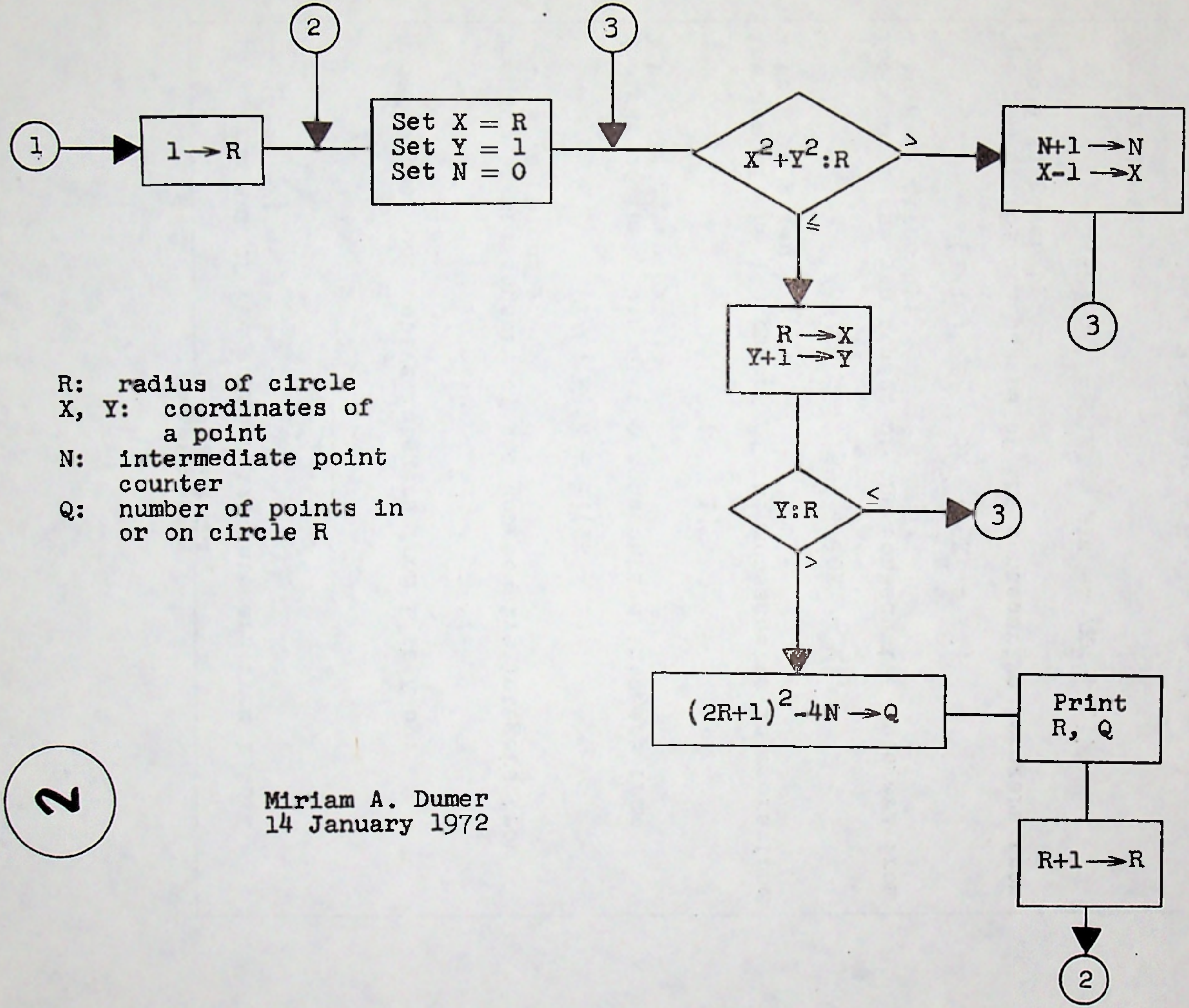
$$X^2 + Y^2 \leq R^2 \qquad (1)$$

Clearly, a direct evaluation of (1) would be inefficient, since it does not capitalize on the symmetry of the problem. Nevertheless, the flowchart (2) shows a straightforward approach to the problem.

Much greater computational efficiencies can be obtained by observing (as Rose's paper does) that the decision involved in (1) is trivial for most of the points within the circle. The points for which computation is needed are those lying close to the circle. By using this idea and other shortcuts, Richard Sandin calculated (1/14/72) the results shown in the following table:

R	Q
10	317
20	1257
30	2821
40	5025
50	7845
100	31417
200	125629
300	282697
400	502625
500	785349
600	1130913
700	1539297
800	2010573
900	2544569
1000	3141549





R: radius of circle
X, Y: coordinates of a point
N: intermediate point counter
Q: number of points in or on circle R

2

Miriam A. Dumer
14 January 1972

A possible solution to the lattice problem of Gauss.

Formulas

The sum of the consecutive integers from 1 to K is given by:

$$\frac{K(K + 1)}{2}$$

and for the consecutive integers from L to K by:

$$(1/2)(K^2 + K - L^2 - L).$$

The sum of the squares of the consecutive integers from 1 to K is given by:

$$(1/6)(K)(K + 1)(2K + 1).$$

The sum of the cubes of the consecutive integers from 1 to k is given by:

$$(1/4)(K^2)(K + 1)^2.$$

The sum of the 4th powers of the consecutive integers from 1 to K is given by:

$$(1/30)(6K^5 + 15K^4 + 10K^3 - K).$$

The sum of the 5th powers of the consecutive integers from 1 to K is given by:

$$(1/12)(2K^6 + 6K^5 + 5K^4 - K^2)$$

The sum of the 6th powers of the consecutive integers from 1 to K is given by:

$$(1/42)(6K^7 + 21K^6 + 21K^5 - 7K^3 + K).$$

For the series

$$1\cdot 3 + 2\cdot 4 + 3\cdot 5 + 4\cdot 6 + 5\cdot 7 + \ldots + K(K + 2)$$

the sum is given by:

$$(1/6)(2K^3 + 9K^2 + 7K).$$

A Way to Sort

PROBLEM 49

CANDY
EVERY
MISTY
HEAVY
GREAT
EDICT
NIGHT
FAULT
JAUNT
PRINT
EVENT
INPUT
ITEMS
DROSS
SUGAR
RADAR
LIVER
OTTER
RULER
BAKER
ADDER
QUEEN
CREAM
LOYAL
FINAL
STEEL
STALL
BREAK
DRINK
ROUGH
YOUNG
WRONG
BRING
DUNCE
PRIDE
KNIFE
UNCLE
GRIME
DRONE
WHOSE
FALSE
WRITE
ELITE
ABOVE
MOVED
TRIED
GRAND
BROOD
UMBRA
ZEBRA

The 50 words on the left are in alphabetic order. The ordering is not the customary one: the major sort is on the last letter, in descending order; the intermediate sort is on the second last letter, in ascending order; the minor sort is on the third last letter, in descending order.

(A) Write a program to accept any number of 5-letter words and output them with the same sorting scheme.

(B) The same scheme has been applied to the 25 words on the right. Modify your program from (A) to accept any number of words of any length greater than two letters and output them resorted the same way.

SATISFY
CAT
BROUGHT
ADAMANT
OUTPUT
ICICLES
DELIVER
CLOISTER
WONDER
TOBACCO
STUDIO
FORTRAN
DECISION
NATION
ALGORITHM
MUSEUM
PRINCIPAL
ASTONISH
STRING
SHERIFF
TIDE
STORAGE
STRIVE
TOLD
BASIC

Sequences of Triangles

PROBLEM 38 SOLUTION

Starting with an equilateral triangle with unit area, a sequence of equilateral triangles is formed in which the area of one is the altitude of the previous one. The problem was to find the altitude of the 100th such triangle.

Several readers pointed out that the sequence converges, so that

$$A_{n+1} = \sqrt[4]{3}\,\sqrt{A_n}$$

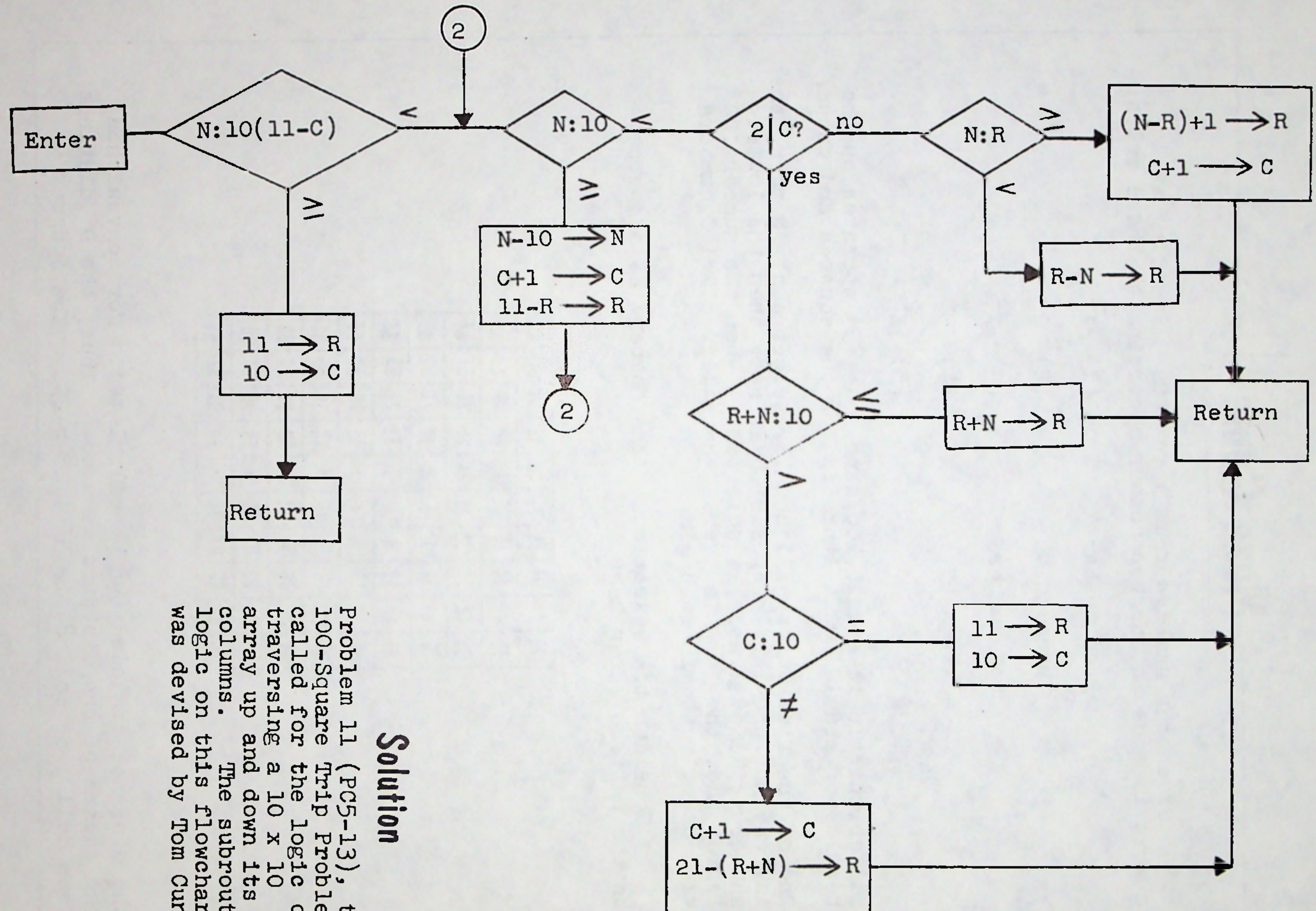
from which it can be deduced that

$$A_\infty = \sqrt{3}.$$

But that wasn't the problem; the problem was to find A_{100}. Associate Editor David Babcock calculated:

$$A_{100} = 1.7320508075688772935274463415051218$$

which agrees with the square root of 3 (see PC3-6) to 31 significant digits.



Solution

Problem 11 (PC5-13), the 100-Square Trip Problem, called for the logic of traversing a 10 x 10 array up and down its columns. The subroutine logic on this flowchart was devised by Tom Cundey.

THE
Maze Game
PROBLEM 50

Given the numbers from 1 to 100. Starting at 1, a move is made to another number according to these rules:

From number X	(1)	X^2
proceed to	(2)	2^X
the number	(3)	X^3
given by:	(4)	3^X
	(5)	$\lvert X^2 - X^3 \rvert$

[all arithmetic is modulo 100, and rule (5) operates on the results of modulo 100 arithmetic for the squares and cubes]

One of the five rules will usually select a new number; that is, one not previously chosen. If not, then the lowest number still available is selected. When the number 100 is reached, the game is over. Normally, all 100 numbers will be selected in each game.

Consider the selection of the numbers as a journey on the pattern shown here:

73	74	75	76	77	78	79	80	81	82
72	43	44	45	46	47	48	49	50	83
71	42	21	22	23	24	25	26	51	84
70	41	20	7	8	9	10	27	52	85
69	40	19	6	1	2	11	28	53	86
68	39	18	5	4	3	12	29	54	87
67	38	17	16	15	14	13	30	55	88
66	37	36	35	34	33	32	31	56	89
65	64	63	62	61	60	59	58	57	90
100	99	98	97	96	95	94	93	92	91

the distance traveled can be calculated. For convenience, the square of the distance is used. Thus, the D^2 distance from cell 1 to cell 84 is $5^2 + 2^2 = 29$. The journey

begins at 00, so the first leg, to cell 1, has a D^2 distance of 50 to start.

The Problem is, what ordering (following the move rules) will produce (A) the longest journey, or (B) the shortest journey? Present records are 2711 for A and 1803 for B. The shortest known journey begins as follows:

1	arbitrary
2	only possible move
8	X^3
12	X^3
96	2^X
20	$\lvert X^3 - X^2 \rvert$
76	2^X
36	2^X
40	difference between powers
3	by rule (6)--by default
27	X^3
54	difference
64	X^3
44	X^3
16	2^X
21	3^X
52	2^X

Solution

Problem 37 (PC12-1), the Sine Excursion trip, called for a 600-leg journey in which the lengths of each leg were given by the decimal expansion of sine 1, and the turns were uniformly one radian clockwise.

Thomas R. Parkin, Control Data Corporation, furnishes these results:

$$X = 32.9678624079$$
$$Y = -70.4643240552$$

N-Series

Log 14	1.146128035678238025925955153317129220251762277786073947814062414845361629176503675553038779965674 75
Ln 14	2.639057329615258614522584864901356297712584863942116442580070159430973484721763983393521825584290 21
$\sqrt{14}$	3.741657386773941385583748732316549301756019807778726946303745467320035156306939027976809895194379 58
$\sqrt[3]{14}$	2.410142264175229986128369667603272895354581289980867654164139710413291726922593833822611516226813 47
$\sqrt[5]{14}$	1.695218203072435481549343584607767115294380564684091593099616358054583236090808177441589003253712 00
$\sqrt[7]{14}$	1.457916249576283530691311271122606934306926764471354252221119466449337925197185565657078460176015 252
$\sqrt[10]{14}$	1.302005454317467704497249303077425630323028891511193539762718482737573775709850991488678735894791 68
$\sqrt[100]{14}$	1.026741888133729235468453639510415944232106263416457619232852601741149291081091094523484414365230 84
e^{14}	1202604.284164776777749236770767859449412486543376102240313290633197462947083342670903641929 64
π^{14}	9122171.181754353170204375110762816274502700883297762252993768387309742763623777951986300834 60
$\tan^{-1} 14$	1.499488862009606292798950701786658381075284768457510831674279832024365652978176830278453026880710 88
14^{100}	410018608884993288052964165246709725458010675237920273221971263567489261466026483061479032219018658198 1413953765376

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